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Spherical Wave Propagation in Water

SHERMAN C. LOWELL

PART I.

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SPHERICAL WAVE PROPAGATION IN WATER

PART I.

by

Sherman C. Lowell

AFSWP-714

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This report represents results obtained
at the Institute of Mathematical Sciences,
New York University, under the auspices
of the Office of Naval Research.

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PREFACE

This is the first of a set of three reports on the theoretical analysis of the shock wave produced by an underwater atomic explosion. The purpose of the analysis is to determine the peak pressure, impulse and arrival time of the shock as a function of distance from the explosion and of the energy released. In addition the pressure and velocity distributions behind the shock will be found.

The various quantities mentioned above have been determined in two different ways. These are

1. Numerical solution of the equations of compressible fluid flow, carried out on the UNIVAC at New York University.
2. Approximate analytical solution of the equations of compressible fluid flow.

The analytical solution is partially described in the present report, and the description will be completed in the next report. The numerical solution will be described in the third report of this set where its results will be compared with the analytical results. It will be shown that the two different methods of solution yield results in good agreement with each other. Thus the possibility of solving this problem by an analytical method will be demonstrated. Therefore it will be possible to solve additional problems of this kind entirely analytically. This will afford a considerable saving in time and money over the numerical method, and

provide a better understanding of the physical phenomena involved.

The first step in both the numerical and analytic methods is the calculation of the "strong shock" solution. This solution desceibes the initial behavior of the shock. Although it has been studied before, it is discussed anew in the present report because of its basic role in both methods. Furthermore various procedures are developed for simplifying the calculation of this solution. First, of course, the problem is formulated and the equation of state of water is discussed. Afterwards the equations are converted to dimensionless form.

The table of contents indicates what the next two reports will contain.

SPHERICAL WAVE PROPAGATION IN WATER

§1. The Equation of State of Water

When the pressure is less than fifty kilobars, a moderately good fit to the experimental pressure-volume-temperature values is given by the relation

$$(1.1) \qquad (p + P)V = BT \ .$$

In order to get the best agreement with experiment it is desirable to make the quantities P and B depend on the temperature, but for the purposes of this discussion we shall assume they are constant. The quantity P is called the internal pressure and is approximately equal to 3000 atmospheres.

The enthalpy of the water is defined by the relation

$$(1.2) \qquad i = e + pV$$

where e is the internal energy. It can be shown from the laws of thermodynamics that the change in enthalpy between two states (p_o, T_o) and (p, T) can be calculated from the relation

$$(1.3) \quad i - i_o = \int_{T_o}^{T} c_p(T', p_o)\,dT' + \int_{p_o}^{p} \left\{ V(p,T) - T \frac{\partial V(p', T)}{\partial T} \right\} dp'$$

where $c_p(T', p_o)$ is the specific heat at constant pressure p .

It is easily shown that, when the state equation has the form (1.1), the second term vanishes identically and we can write

$$(1.4) \qquad i - i_o = \bar{c}_p(T - T_o)$$

where $\bar{c}_p$ is the average value of $c_p(T', p_o)$ in the range $T_o \leq T' \leq T$. An equivalent form of (1.4) is

$$(1.5) \qquad i - i_o = \frac{\gamma}{\gamma - 1} \left\{ (p + P)V - (p_o + P)V_o \right\}$$

where we have introduced the ratio of specific heats at constant pressure and constant volume, $\gamma = \bar{c}_p / \bar{c}_v$, and the thermodynamic fact that $B = \bar{c}_p - \bar{c}_v$. For water γ has an average value of 7.15 and $\bar{c}_p$ is approximately equal to 3.9644 joules/gm.deg.

The expression for the internal energy is easily computed:

$$(1.6) \qquad e - e_o = \frac{1}{\gamma - 1} [(p+P)V - (p_o+P)V_o] + P(V - V_o)$$

We also compute, from the first law of thermodynamics, the entropy change

$$(1.7) \quad S - S_o = \int_{T_o}^{T} c_p \frac{dT'}{T'} - \int_{p_o}^{p} V(p', T) \frac{dp'}{T} = c_p \ln \frac{T}{T_o} - B \ln \frac{p+P}{p_o+P} \quad .$$

From this expression and (1.1) we derive the entropic form of the state equation:

$$(1.8) \qquad (p+P)V^{\gamma} = (p_o+P)V_o^{\gamma} \exp\left(\frac{S-S_o}{\bar{c}_v}\right) \quad .$$

The sound speed c in water is easily obtained from its definition by use of (1.8). It is given by

$$(1.9) \qquad c^2 = -V^2\left(\frac{\partial p}{\partial V}\right)_S = \gamma(p+P)V = \gamma BT \quad .$$

§2. The Equations of Fluid Motion

It is convenient in discussing spherical blast waves to introduce a Lagrangean coordinate h which is proportional to the mass contained inside a sphere of radius r. We define

$$(2.1) \qquad h = \int_o^{r(h,t)} \frac{1}{V} r^2 dr$$

which in differential form, using subscript notation for differentiation,

$$(2.2) \qquad V = r^2 r_h$$

expresses the law of mass conservation.

Newton's law of momentum conservation can then be written

$$(2.3) \qquad u_t = -r^2 p_h \quad .$$

We shall, moreover, assume that all motions behind a hydrodynamic shock take place adiabatically, so that the law of energy

conservation can be written in the equivalent forms

(2.4a) $$S_t = 0$$

or

(2.4b) $$Vp_t + \gamma(p + P)V_t = 0 \quad .$$

To make this system of partial differential equations complete we must add the kinematic equation

(2.5) $$r_t = u$$

and one of the state equations (1.1) or (1.8) .

§3. The Shock Conditions

If a shock or pressure discontinuity exists in the water medium, then, as a consequence of the conservation laws, the states on the two sides of the shock are connected by the Rankine-Hugoniot conditions, which may be written

(3.1) $$V_o(U - u_1) = V_1 U$$

(3.2) $$V_o(p_1 - p_0) = u_1 U$$

$$(3.3) \qquad i_1 - i_0 = (1/2)(V_0 + V_1)(p_1 - p_0) \quad .$$

In these equations the subscript 1 refers to the state just behind the shock and the subscript 0 refers to the ambient or rest state ahead of the shock. We assume that the water is at rest ahead of the shock. The quantity $U = U(t)$ is the shock speed and we have the relation

$$(3.4) \qquad U = dR/dt$$

where $r = R(t)$ is the position of the shock at time t.

It is convenient for our purposes to transform the equations (3.1) to (3.3) by use of the results of §1. In addition we define the shock pressure ratio

$$(3.5) \qquad \xi_1 = \frac{p_1+P}{p_0+P} \quad .$$

Then the shock conditions can be put in the form

$$(3.6) \qquad V_1 = V_0(\mu^2 \xi_1 + 1)(\xi_1 + \mu^2)^{-1}$$

$$(3.7) \qquad u_1 = U(1 - \mu^2)(\xi_1 - 1)(\xi_1 + \mu^2)^{-1}$$

$$(3.8) \qquad p_1 + P = U^2(1 - \mu^2)\xi_1(\xi_1 + \mu^2)^{-1}V_0^{-1}$$

where $\mu^2 = (\gamma-1)/(\gamma+1)$.

In particular we are interested in the limiting forms of (3.5) - (3.8) when the shock strength becomes very large. The resulting relations are known as the strong shock conditions:

$$V_1 = \mu^2 V_o \tag{3.9}$$

$$u_1 = (1 - \mu^2)U \tag{3.10}$$

$$p_1 + P = (1 - \mu^2)U^2/V_o \quad . \tag{3.11}$$

§4. Taylor Waves

An important type of solution of the hydrodynamic equations and boundary conditions has been discussed by G. I. Taylor [1]. The solutions are also variously called progressing waves, strong shock solutions, point blast solutions, or similarity solutions, cf. [2,3,4]. They may be described in the following way. Let $r = R(t)$ denote the shock path and $U(t) = dR/dt$ the shock velocity. We then ask whether the Euler radius of a particle $r(h,t)$, the particle velocity $u(h,t)$, the specific volume $V(h,t)$ and the pressure $p(h,t)$ can be represented as a product of a function of the time by a function of a similarity variable x, which depends on both h and t in some prescribed way. For our purposes it is

convenient to make the choice

$$r(h,t) = R(t)f(x,\gamma) \tag{4.1}$$

$$u(h,t) = (1-\mu^2)U(t)g(x,\gamma) \tag{4.2}$$

$$V(h,t) = \mu^2 V_o \tau(x,\gamma) \tag{4.3}$$

$$p(h,t) + P = (1 - \mu^2)V_o^{-1}U^2(t)\pi(x,\gamma) \tag{4.4}$$

where $x = h/H(t)$ and $H(t) = R^3(t)/3V_o$.

In order to understand the restrictions that are implicit in this form of solution we shall examine the energy balance in the blast wave. Since the motions behind the shock are assumed to take place adiabatically, the total energy, internal plus kinetic, of the fluid inside the shock sphere at any instant must equal the yield energy Q put into the blast plus the internal energy of the ambient fluid taken into the shock sphere, i.e.

$$Q = 4\pi H(t) \int_0^1 (e - e_o + \tfrac{1}{2}\mu^2)dx \quad . \tag{4.5}$$

But, with our assumptions about the solution,

$$e-e_o + \frac{1}{2}u^2 = \frac{(1-\mu^2)^2}{2} U^2(t)[\pi(x,\gamma)\tau(x,\gamma)+ g^2(x,\gamma)]$$

$$+ PV_o[\mu^2\tau(x,\gamma)-1] - \frac{(1-\mu^2)}{2\mu^2}(p_o + P)V_o$$

and hence

$$Q = 2\pi(1-\mu^2)^2H(t)U^2(t)I(\gamma) - 2\pi\mu^{-2}(1-\mu^2)(p_o+P)V_oH(t)$$

where we have set

$$(4.6) \qquad I(\gamma) = \int_o^1 [\pi(x,\gamma)\tau(x,\gamma) + g^2(x,\gamma)]dx$$

and have made use of the fact, which follows from (2.2), that

$$\int_o^1 [\mu^2\tau(x,\gamma) - 1]dx = 0 \quad .$$

If we modify the form of the last term by using the relations (3.5) and (3.8), the expression for Q becomes

$$(4.7) \quad Q = 2\pi(1-\mu^2)^2I(\gamma)H(t)U^2(t)[1-1/\mu^2I(\gamma)(\xi_1(t)+\mu^2)] \quad .$$

Since the yield Q is independent of the time, the expression on the right hand side must be also. This can certainly be arranged by setting

(4.8) $$U^2(t)R^3(t) = A^2$$

where A is a constant and by taking $\xi_1(t)$ so large that the second term is essentially zero. The first requirement implies that

(4.9) $$R^5(t) = \frac{25}{4} A^2 t^2$$

and the second implies that our solution will only be valid when the shock pressure ratio is very large. The appropriate boundary conditions for the Taylor waves are therefore the strong shock relations (3.9) - (3.11).

When the last term in the bracket of equation (4.7) is set equal to zero, the constant A can be determined in terms of the yield Q, the ambient specific volume V_o , and the adiabatic exponent γ by means of the relation

(4.10) $$A^2 = 3V_oQ/2\pi(1 - \mu^2)^2 I(\gamma) \quad .$$

§5. Scaling of the Equations to Dimensionless Form

We can, of course, measure the physical quantities involved in our results so far in any consistent dimension system. There is a choice of units, however, which is related to the parameters of the problem and which leads to results of much greater mathematical simplicity. This possibility depends on the fact that all explosion phenomena are similar, if the motion is adiabatic behind the shock, by which we mean that they depend only on the initial energy or yield of the blast Q and upon the quantities characterizing the ambient state ahead of the shock, for example the pressure p_o and specific volume V_o. We shall choose our dimension units so as to make our results independent of these quantities. They are as follows:

Length unit: λ, where $\lambda^3 = Q/p_o+P$

Velocity unit: c_o, where $c_o^2 = \gamma(p_o+P)V_o$

Time unit: λ/c_o

Pressure unit: $p_o + P$

Energy unit: c_o^2

Mass unit: λ^3/V_o

Specific volume unit: V_o

Our previous results can then be restated in the form

$$i = \frac{1}{\gamma-1}\,\xi V \qquad \text{(enthalpy)} \tag{5.1}$$

where $\xi = (p + P)/(p_o + P)$ is the pressure ratio.

(5.2) $e = \gamma^{-1}(\frac{1}{\gamma-1}\,\xi + k)V$ (internal energy)

where $k = P/(p_o + P)$.

(5.3) $\xi V^{\gamma} = \exp S_*$ (entropic state equation)

where $S_* = (S-S_o)/c_v$.

(5.4) $c^2 = \xi V = B_* T$ (sound speed)

where $B_* = \gamma(\gamma - 1)c_v/c_o^2$.

(5.5) $V = r^2 r_h$ (continuity equation)

(5.6) $r_t = u$ (Lagrangean velocity definition)

(5.7) $u_t = -\gamma^{-1} r^2 \xi_h$ (momentum equation)

(5.8) $V\xi_t + \gamma\xi V_t = 0$ (adiabatic energy equation)

(5.9) $V_1(\ _1 + \mu^2) = \mu^2\ _1 + 1$ (first shock condition)

(5.10) $u_1(\xi_1 + \mu^2) = U(1 - \mu^2)(\xi_1 - 1)$ (second shock condition.

(5.11) $\xi_1 + \mu^2 = (1 + \mu^2)U^2$ (third shock condition)

(5.12) $V_1 = \mu^2$ (first strong shock condition)

(5.13) $u_1 = (1 - \mu^2)U$ (second strong shock condition)

(5.14) $\xi_1 = (1 + \mu^2)U^2$ (third strong shock condition) .

In order to get a feeling for the magnitude of the quantities involved we note that if we take $p_o + P$ to be 3000 bars, the unit of length λ will then be 10 meters if the yield is 71.6 metric tons of TNT and will be 100 meters if the yield is 71.6 metric kilotons of TNT or its equivalent. If the yield is 20 kilotons of TNT equivalent, the length unit is 65.3 meters.

§6. The Solution of H. Primakoff for $\gamma = 7$.

If the equation of state has the form (1.8) with $\gamma = 7$ and $\mu^2 = 3/4$ an explicit solution of the dynamical equations (5.5) - (5.8) can be given cf. [2,4], which is compatible with the strong shock conditions (5.12) - (5.14). (This special solution was also discovered earlier by VanVleck but its significance for water blasts was not noticed.) It is a special case of the progressing wave or point blast solution discussed by G. I. Taylor. As a preliminary we compute the total mass of water $\bar{H}$ which is contained inside the shock sphere at time t. Aside from a factor 4π it is given by

$$\bar{H}(t) = \int_0^{\bar{R}(t)} \bar{V}^{-1}\bar{r}^2 d\bar{r} = \int_0^{\bar{R}(t)} r_o^2 dr_o$$

where r_o is the initial distance of a particle from the center and the second relation follows from the equation of mass conservation. But in the latter form the integration can be performed explicitly with the result

$$\bar{H}(t) = \bar{R}^3/3 \quad . \tag{6.1}$$

This equation is independent of the value of γ .

The Primakoff solution for $\gamma = 7$ can be written in the form

$$\bar{r}(h,t) = \bar{R}(t)\bar{x}^{0.25} \tag{6.2}$$

$$\bar{u}(h,t) = 0.25\bar{U}(t)\bar{x}^{0.25} \tag{6.3}$$

$$\bar{V}(h,t) = 0.75\bar{x}^{-0.25} \tag{6.4}$$

$$\bar{\xi}(h,t) = 7/4\bar{U}^2(t)\bar{x}^{3/4}\ 1.75\bar{U}^2(t)\bar{x}^{0.75} \tag{6.5}$$

where

$$\bar{x} = h/\bar{H}(t) \quad . \tag{6.6}$$

The values of these quantities just behind the shock are found by setting $\bar{x} = 1$.

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It is also instructive to write these quantities as functions of the Euler distance r. We define

$$\bar{y} = \bar{r}/\bar{R}(t) \tag{6.7}$$

and then we have

$$h(r,t) = \bar{H}(t)\bar{y}^4 \tag{6.8}$$

$$\bar{u}(\bar{r},t) = 0.25\ \bar{U}(t)\bar{y} \tag{6.9}$$

$$\bar{V}(\bar{r},t) = 0.75\ \bar{y}^{-1} \tag{6.10}$$

$$\bar{\xi}(\bar{r},t) = 1.75\ \bar{U}^2\bar{y}^3 \quad . \tag{6.11}$$

These relationships are shown in Figure 1.

The shock speed and radius are determined since $I(\gamma=7) = 4/3$ by application of (4.6), from the relations

$$\bar{U}(t) = (18/7\pi)^{0.5}\bar{R}^{-1.5} \tag{6.12}$$

$$\bar{R}(t) = (225/14\pi)^{0.2}t^{0.4} \quad . \tag{6.13}$$

The following formulae, which we make use of later, follow from (6.1), (6.12) and (6.13):

(6.14) $$\bar{H}^{-1} d\bar{H}/dt = 3\bar{R}\,\bar{U}^{-1} \quad \text{or} \quad d\bar{H}/dt = \bar{R}^2\bar{U}$$

(6.15) $$d\bar{U}/dt = 1.5\,\bar{R}^{-1}\bar{U}^2 \quad .$$

The use of the results derived in this section may be illustrated by the following example. Suppose we know that for a certain explosion in water the total energy going into the blast wave was the equivalent of 10 metric kilotons of TNT. We wish to find the conditions existing at the time the shock sphere has a radius of 20 meters.

We may assume without making any significant error that water has the value of $\gamma = 7$ and $p_o + P = 3000$ bars. In order to find the unit of distance we use the relation

$$\lambda = \sqrt[3]{Q/p_o+P} = \sqrt[3]{10\text{x}4.185\text{x}10^{19}\,\text{dyns-cm}/3.000\text{x}10^{6}\,\text{dyne/cm}^2}$$

$$= \sqrt[3]{1.395\text{x}10^{11}\,\text{cm}^3} = \sqrt[3]{139.5\text{x}10^{3}}\ \text{meters} = 51.8\ \text{meters}$$

Hence, the dimensionless shock radius is $\bar{R} = 20/51.8 = 0.386$. The sound speed in the ambient region is

$$c_o = \sqrt{\gamma V_o (p_o+P)} = \sqrt{7\text{x}1\text{cm}^3/\text{gmx}3\text{x}10^{9}\,\text{dynes/cm}^2} = \sqrt{2.1}\ \text{x}\ 10^{3}\ \text{m/sec}$$

$$= 1450\ \text{m/sec} \quad .$$

From equation (6.12) we compute the dimensionless shock speed, $\bar{U} = \sqrt{18/7\pi\bar{R}^3} = \sqrt{18/7\pi(0.382)^3} = 3.79$. The actual shock speed is 3.79 x 1450 = 5500 m/sec.

From equation (6.11) with $\bar{y} = 1$, we can compute the dimensionless shock pressure ratio, $\xi_1 = 1.75\ \bar{U}^2 = 25.2$. (If this value had been less than 10, the use of these simplified formulas would not be justified.) The actual pressure at the shock is $p = 25.2 \times 3000 - 3000$ bars or $p = 72.6$ kilobars, assuming that p_o is approximately zero. The pressure at a point 10 meters from the center of the shock sphere is found from (6.7) and (6.11): $y = \bar{r}/\bar{R} = r/R = 10/20 = 0.5$, and $\bar{\xi}(0.5,t) = 25.2 \times (0.5)^3 = 3.15$ or $p = 6.45$ kilobars.

It is perhaps appropriate at this point to say something about the range of applicability of the results obtained so far. Two principal simplifying assumptions have been made, one hydrodynamic in nature and the other thermodynamic.

The chief feature of the hydrodynamic simplification was the restriction to strong shocks. Thus we neglected consistently terms of order $1/\xi_1$. A crude estimate would therefore put the error due to this assumption in the same order of magnitude. If, for example, a maximum error of ten percent is allowable we should not use the Primakoff solution when the pressure behind the shock has dropped below about 30,000 atmospheres.

The errors due to the thermodynamic assumption that the equation has the simple Tait form (1.8) are much more difficult to

gauge, since our knowledge of the thermodynamic properties of water is very meager for high pressures and temperatures. In one respect, however, it is possible to estimate the effect of a change in the equation of state. It will be recalled that the Primakoff solution holds only for the value $\gamma = 7$, while the average value of γ for water was stated earlier to be about 7.15 .

In the succeeding sections we will show how the Primakoff solution can be modified by perturbation procedures to give the solution when the shock strength is not large and when γ is different from the value seven.

§7. The Small γ-7 Approximation

In order to improve the Primakoff solution of the previous section, so that it holds for values of γ in the neighborhood of 7, we return to the general equations of fluid motion (5.5) - (5.8) and the corresponding strong shock relations (5.12) - (5.14). We replace γ, whenever it occurs in the equations, by $7 + \varepsilon$ and make the assumption that for small enough values of ε the quantities r, u, V and p have a convergent power series expansion in ε of which the first term is the Primakoff solution. Moreover, we look for progressing wave or similarity type solutions in which the higher order terms in the ε-expansion depend on h and t in the same way as in the Primakoff solution, i.e. they can be written as the product of a function of the time by a function of x, where $x = h/H$ and H denotes the mass of the perturbed shock sphere. More precisely, we set

$$
\begin{aligned}
r(h,t) &= \bar{r} + \varepsilon\bar{R}\, x^{0.25} f(x) = \bar{R}(t)x^{0.25}\left\{(R/\bar{R})^{0.75} + \varepsilon f(x)\right\} \\
u(h,t) &= \bar{u} + 0.25\varepsilon\bar{U}x^{0.25} g(x) = 0.25\bar{U}(t)x^{0.25}\left\{(R/\bar{R})^{0.75} + \varepsilon g(x)\right\} \\
V(h,t) &= \bar{V} + 0.75\varepsilon x^{-0.25}\tau(x) = 0.75^{-0.25}\left\{(R/\bar{R})^{2.25} + \varepsilon\tau(x)\right\} \\
\xi(h,t) &= \bar{\xi} + 1.75\varepsilon\bar{U}^2 x^{0.75}\pi(x) = 1.75\bar{U}^2(t)x^{0.75}\left\{(R/\bar{R})^{2.25} + \varepsilon\pi(x)\right.
\end{aligned}
\tag{7.1}
$$

where we have retained only two terms in the ε-series expansion.

But, since $R(t) = r(H,t)$ and $U = dR/dt$, we have

$$
R/\bar{R} = (R/\bar{R})^{0.75} + \varepsilon f(1) = 1 + 4\varepsilon f(1) = U/\bar{U} \quad . \tag{7.2}
$$

Consequently, relations (7.1) may be written

$$
r(x,t) = R(t)x^{0.25}\left\{1 + \varepsilon[f(x) - f(1)]\right\} \tag{7.3}
$$

$$
u(x,t) = 0.25U(t)x^{0.25}\left\{1 + \varepsilon[g(x) - f(1)]\right\} \tag{7.4}
$$

$$
V(x,t) = 0.75x^{-0.25}\left\{1 + \varepsilon[\tau(x) - 3f(1)]\right\} \tag{7.5}
$$

$$
\xi(x,t) = 1.75U^2(t)x^{0.75}\left\{1 + \varepsilon[\pi(x) + f(1)]\right\} \quad . \tag{7.6}
$$

The values of these quantities just behind the shock may be obtained by setting $x = 1$ in (7.4) - (7.6), so that

$$(7.7)\qquad u_1(t) = 0.25U(t)\left\{1 + \varepsilon[g(1) - f(1)]\right\} = (1 - \mu^2)U(t)$$

$$(7.8)\qquad V_t(t) = 0.75\left\{1 + \varepsilon[\tau(1) - 3f(1)]\right\} = \mu^2$$

$$(7.9)\qquad \xi_1(t) = 1.75\, U^2(t)\left\{1 + \varepsilon[\pi(1) + f(1)]\right\} = 7(1 - \mu^2)U^2(t)\ ,$$

where the last equality is just the statement of the strong shock relations. Since, by a simple calculation,

$$(7.10)\qquad \mu^2 = 0.75(1 + \varepsilon/24),\ (1 - \mu^2) = 0.25(1 - \varepsilon/8)$$

we get, by comparison, the boundary relations

$$(7.11)\qquad \begin{aligned} \tau(1) - 3f(1) &= 1/24 \\ g(1) - f(1) &= -1/8 \\ \pi(1) + f(1) &= -1/8 \quad . \end{aligned}$$

Similarly the insertion of (7.3) - (7.6) into the equations of fluid motion (5.5) - (5.8) gives the differential equations satisfied by the perturbation functions

$$(7.12)\qquad \tau(x) - 3f(x) = 4xf'(x)$$

$$(7.13)\qquad g(x) - f(x) = -12xf'(x)$$

$$(7.14)\qquad 3\pi(x) + 4x\pi'(x) = 3g(x) + 4xg'(x) - 6f(x)$$

$$4x\pi'(x) = 1 - 28x\,\tau'(x) \quad . \tag{7.15}$$

The last of these equations can be integrated immediately to yield

$$\pi(x) + 7\tau(x) = \frac{1}{4}\ln x + 20\,f(1) + \frac{1}{6} \tag{7.16}$$

which combines with the other equations to give a single second order equation for the function $f(x)$:

$$16x^2 f''(x) + 50xf'(x) + 15f(x) = \frac{3}{16}\ln x + 15f(1) + \frac{3}{8} \tag{7.17}$$

whose solution, subject to the boundary conditions (7.11), is

$$f(x) - f(1) = \frac{1}{80}\ln x + \frac{1}{300}x^{-5/8} - \frac{1}{300} \quad . \tag{7.18}$$

From this solution we obtain very easily

$$\tau(x) - 3f(1) = \frac{3}{80}\ln x + \frac{1}{600}x^{-5/8} + \frac{1}{25} \tag{7.19}$$

$$g(x) = f(1) = \frac{1}{80}\ln x + \frac{17}{600}x^{-5/8} - \frac{23}{150} \tag{7.20}$$

$$\pi(x) + f(1) = -\frac{1}{80}\ln x - \frac{7}{600}x^{-5/8} - \frac{17}{150} \quad . \tag{7.21}$$

The solution is completed by computing the relation between U and R. This is done most easily by use of the results of §4. In particular we use equations (4.8), (4.10) and (4.6) to obtain the

result that

$$U^2R^3 = A^2 = \frac{18}{7\pi}(1 + \varepsilon/4)t \tag{7.22}$$

from which it follows that

$$R^{2.5}(t) = \frac{15}{\sqrt{14\pi}}(1 + \varepsilon/8)t \tag{7.23}$$

A further by product is the evaluation $f(1) = 1/80$, although this result is only useful for a comparison of the Taylor solutions for $\gamma = 7$ and $\gamma = 7 + \varepsilon$.

The final expressions for r, u, V and ε as functions of the variables x and t can be written

$$r/R(t) = x^{0.25}[1 + \frac{\varepsilon}{20}(\frac{1}{4}\ln x + \frac{1}{15}x^{-0.625} - \frac{1}{15})] \tag{7.24}$$

$$u/U(t) = \frac{1}{4}x^{0.25}[1 + \frac{\varepsilon}{20}(\frac{1}{4}\ln x + \frac{17}{30}x^{-0.625} - \frac{46}{15})] \tag{7.25}$$

$$V = \frac{3}{4}x^{-0.25}[1 + \frac{\varepsilon}{20}(\frac{3}{4}\ln x + \frac{1}{30}x^{-0.625} + \frac{4}{5})] \tag{7.26}$$

$$\varepsilon/U^2 = \frac{7}{4}x^{0.75}[1 - \frac{\varepsilon}{20}(\frac{1}{4}\ln x + \frac{7}{30}x^{-0.625} + \frac{34}{15})] \quad . \tag{7.27}$$

For the understanding and application of these results it is, however, more convenient to express the results as functions of the Euler radius r and the time . The inversion of (7.24) - (7.27) in terms of the variable $y = r/R(t)$ gives

$$(7.28) \qquad x = h/H(t) = y^4[1 - \frac{\varepsilon}{5}(\ln y + \frac{1}{15} y^{-2.5} - \frac{1}{15})]$$

$$(7.29) \qquad u/u_1(t) = y[1 + \frac{\varepsilon}{40}(y^{-2.5} - 1)]$$

$$(7.30) \qquad V/V_1 = y^{-1}[1 + \frac{\varepsilon}{20}(4 \ln y + \frac{1}{10} y^{-2.5} - \frac{1}{10})]$$

$$(7.31) \qquad \xi/\xi_1(t) = y^3[1 - \frac{\varepsilon}{20}(4 \ln y + \frac{13}{30} y^{-2.5} - \frac{13}{30})] \quad .$$

The values at the shock are given in (7.7) - (7.10).

The terms multiplied by ε in the relations (7.28) - (7.31) are plotted in Figure 2. The corrections are probably valid even for moderately large ε within the outer half of the shock sphere but should not be taken too seriously in the neighborhood of the center.

By using the last of equations (7.1) it is very simple to calculate the effect of a change of γ on the shock pressure curve. Thus

$$\begin{aligned} \xi_1(t;\gamma)/\,\bar{\xi}_1(t;7) &= (R/\bar{R})^{2.25} + \varepsilon\pi(1) \\ &= 1 + \left(\tfrac{5}{2}f(1) + \pi(1)\right)(\gamma - 7) \\ &= 1 + (\tfrac{1}{20} - \tfrac{1}{8})(\gamma - 7) = 1 + \tfrac{3}{40}(7 - \gamma) \quad . \end{aligned}$$

Hence, we have, for example,

$$\xi_1(t;5) = 1.15\ \bar{\xi}_1(t;7)$$
$$\xi_1(t;6) = 1.075\ \bar{\xi}_1(t;7)$$
$$\xi_1(t;8) = 0.925\ \bar{\xi}_1(t;7)$$
$$\xi_1(t;7.15) = 1.013\ \bar{\xi}_1(t;7) \quad .$$

It should be emphasized that the results above apply to equal <u>times</u> on the shock pressure curve. If the comparison is made at the same shock <u>radii</u>, the shock pressure ratio ξ_1 for γ near 7 differs by a factor $(1 + \frac{\gamma-7}{8})$ from the Primakoff pressure ratio, i.e., it is greater when $\gamma > 7$ and less when $\gamma < 7$. This is just the opposite of the results shown above and is explained by the fact that the shock radius versus time curve is affected by the change in γ, as is shown by formula (7.2), which can be written

$$R(t;\gamma)/\bar{R}(t;7) = 1 + \frac{\gamma-7}{20} \quad .$$

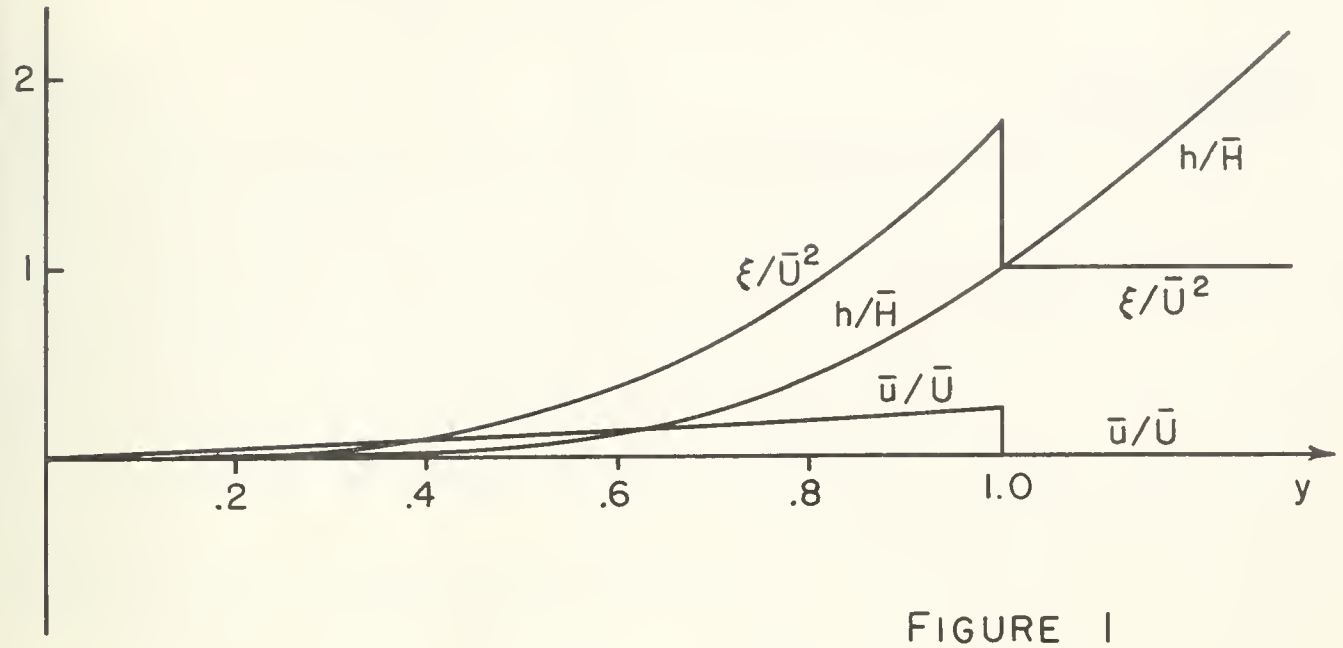

FIGURE 1

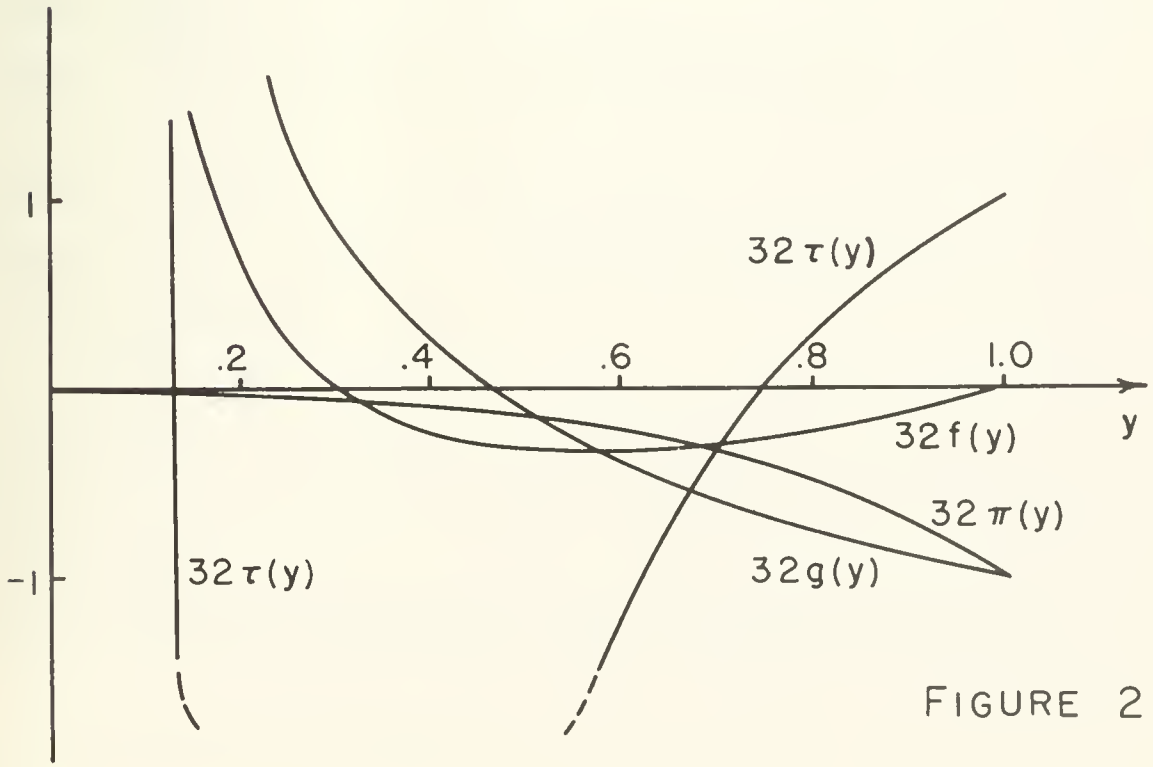

FIGURE 2

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